

# Two loop radion correction to $K_L - K_S$ mass difference in the stabilised Randall-Sundrum brane world scenario

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## Abstract

In the stabilised Randall-Sundrum brane world scenario, the radion can have phenomenologically testable effects, which can be measured against precisely measured electroweak physics data. We investigate the effect of two loop radion corrections to  $K_L - K_S$  mass difference to set a bound on the radion mass and vacuum expectation value. It is found that the leading two loop corrections are of the order  $\left[ \text{Log} \left( \frac{\Lambda^2}{m_\phi^2} \right) \right]^2$  where  $\Lambda$  is the cut-off scale  $\mathcal{O}(\sim \text{TeV})$  and  $m_\phi$  is the radion mass.

**Keywords:** Two loop, Radion Phenomenology,  $K_L - K_S$  mass difference

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## Introduction

Recently, the idea that the fundamental scale of quantum gravity could be dramatically low ( $\mathcal{O} \sim \text{TeV}$ ) as opposed to the Planck scale, if the Standard Model (SM) fields lie on a 3-brane while gravity propagates in the bulk, has gained a lot of attention [1]- [2]. This is yet another attempt to solve the hierarchy problem that plagues the SM. Arkani-Hamed, Dimopoulos and Dvali [1] proposed the existence of  $n$  additional large compact spatial dimensions in which gravity propagates while the SM fields are confined to 3-brane. The usual four dimensional Planck scale ( $M_{pl}$ ) is related to the the effective Planck scale in the bulk (the String scale,  $M_s \sim \mathcal{O}(\text{TeV})$ ) and the volume of the compactified space ( $V_n$ ) as

$$M_{pl}^2 \sim V_n M_s^{n+2} \quad (1)$$

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The bulk space-time is the direct product of four dimensional Minkowski space and the compact space, thus resulting in the relation as in equation(1). Thus, a large value of  $V_n$  may considerably lower the effective scale of quantum gravity, making it possible to test the predictions of such a theory. The imposed requirement of the radius of extra dimensions being large makes such a scenario less attractive. Randall and Sundrum [2] suggested a different picture in which the extra dimensions are small but the background metric is not flat. In the Randall- Sundrum scenario, the SM fields live on one of the two 3-branes in a five dimensional non-factorizable bulk space-time (a slice of  $AdS_5$  space). The metric is

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2 \quad (2)$$

where  $x^\mu$  are the 4-dimensional coordinates while  $y \in [-\pi, \pi]$  parameterizes a  $S^1/Z_2$  orbifold.  $r_c$  is the radius and  $k^{-1}$  is the curvature radius of the  $AdS_5$  space. The two branes lie at  $y = 0$  and  $y = \pi$  with the later brane identified as the ‘visible’ brane.

The geometry of the embedding space-time introduces an exponential warp factor, resulting in a hierarchy between mass scales on the two branes, lowering the natural scale of quantum gravity on the so called ‘visible’ brane, which happens to be our universe. The quantum fluctuations of the inter-brane separation (the radius,  $r_c$ , of the extra dimension) manifest themselves in the form of a scalar particle, the radion, whose mass can be quite low, making it possible for it to have a phenomenologically viable impact, even at low energies. The issue of stabilizing this inter-brane distance or modulus, is an important one. Goldberger and Wise suggested a simple mechanism of stabilising this radius wherein, an additional bulk scalar field coupling to both the branes, is employed [3], providing a nontrivial potential for the radion, consequently stabilizing it. In such a stabilizing scenario, the radion field has a mass which is smaller than the lowest lying Kaluza-Klein mode of the graviton.

The induced metric on the visible brane is

$$g_{\mu\nu}^{ind}(x) = e^{-\frac{\phi(x)}{\langle\phi\rangle}} g_{\mu\nu}(x) \quad (3)$$

where  $\phi$  is the radion field,  $\langle\phi\rangle$  is it’s vacuum expectation value (VEV) and  $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$  gives a tower of massive Kaluza-Klein gravitons as fluctuations about the flat Minkowski metric  $\eta_{\mu\nu} = diag(1, -1, -1, -1)$ , with the zero mass mode coupling to the SM fields with the usual gravitational strength and the massive modes coupling with a strength comparable to the weak scale.

As far as only radion corrections are concerned, the induced metric takes the

form

$$g_{\mu\nu}^{ind}(x) = e^{-\frac{\phi(x)}{\langle\phi\rangle}} \eta_{\mu\nu} \quad (4)$$

The matter action on the visible brane takes the form

$$S_{matter} = \int d^4x \sqrt{-g_{ind}} \mathcal{L}_{SM} \quad (5)$$

with  $\mathcal{L}_{SM}$  being the SM Lagrangian in this warped background. At the moment, one can lay bounds on radion parameters from precisely measured quantities like the electroweak oblique parameters [4] and unitarity constraints arising from gauge boson scattering [5]. In this paper, we intend to constrain the radion parameters using  $K_L$ - $K_S$  mass difference data.

## Radion Coupling to the SM fields

We derive the relevant Feynman rules for radion interacting with the SM particles from the first principles. We first consider for simplicity a simple gauge theory with a fermion and a massive gauge boson (both having acquired mass through Higgs mechanism) in the above warped background. Using the vierbein formalism for the fermions, the Lagrangian for the interaction of the radion with the fermion and the gauge boson is

$$\begin{aligned} \mathcal{L} = & e^{\frac{-3\phi}{2\langle\phi\rangle}} (\iota \bar{\psi} \gamma^\mu \partial_\mu \psi + g \bar{\psi} \gamma^\mu A_\mu \psi) - e^{\frac{-2\phi}{\langle\phi\rangle}} (m \bar{\psi} \psi) \\ & - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + e^{\frac{-\phi}{\langle\phi\rangle}} \left( \frac{1}{2} M^2 A^\mu A_\mu \right) \end{aligned} \quad (6)$$

where  $g$  is the gauge coupling. In deriving the Feynman rules, one has to remember that it is the interaction Hamiltonian and not the Lagrangian that enters the S-matrix. The presence of a warp factor in the kinetic part of the fermion changes the canonical momentum from the usual flat background form by a multiplicative factor  $e^{\frac{-3\phi}{2\langle\phi\rangle}}$ . This results in the interaction Hamiltonian being different from that obtained by naively replacing  $\mathcal{H}_{int}$  by  $-\mathcal{L}_{int}$  for the fermion (see Appendix for details).

If instead, the fermion field is rescaled as

$$\psi \longrightarrow e^{\frac{-3\phi}{4\langle\phi\rangle}} \psi \quad (7)$$

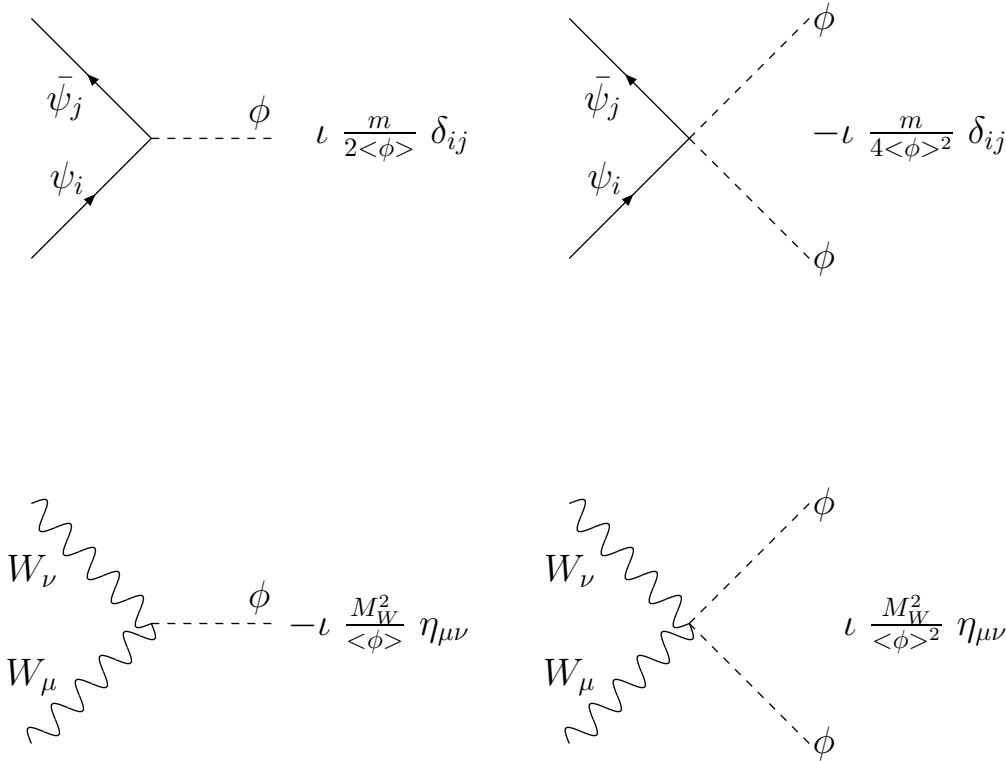
the Lagrangian, in terms of this new field, reads

$$\begin{aligned} \mathcal{L} = & \iota \bar{\psi} \gamma^\mu \partial_\mu \psi + g \bar{\psi} \gamma^\mu A_\mu \psi - e^{\frac{-\phi}{2\langle\phi\rangle}} (m \bar{\psi} \psi) \\ & - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + e^{\frac{-\phi}{\langle\phi\rangle}} \left( \frac{1}{2} M^2 A^\mu A_\mu \right) \end{aligned} \quad (8)$$

Now that there are no derivative terms involved and the kinetic energy term appears in the canonical form,

$$\begin{aligned}\mathcal{H}_{int} &= -\mathcal{L}_{int} \\ &= e^{\frac{-\phi}{2\langle\phi\rangle}} (m\bar{\psi}\psi) - e^{\frac{-\phi}{2\langle\phi\rangle}} \left( \frac{1}{2} M^2 A^\mu A_\mu \right) - g \bar{\psi} \gamma^\mu A_\mu \psi\end{aligned}\tag{9}$$

Clearly, there is no fermion-fermion-gauge boson-radion vertex which appears in other similar calculations [6]. The radion thus couples only to the mass terms and not to the terms involving momenta. The generalization to SM is straight forward and gives the following Feynman rules



where i, j are the fermion flavour indices and m the fermion mass.

The radion, therefore, seems to couple just like the SM Higgs up to first order in the relevant coupling. However, the underlying gauge symmetry of SM regulates the ultraviolet behaviour of radiative corrections due to the Higgs. This feature is absent in the radion interactions, giving substantial contributions to

the otherwise suppressed radiative processes. Moreover, the presence of new vertices (as opposed to the SM Higgs) results in extra diagrams relevant at the desired order in coupling.

## Corrections to $\Delta m_K$

The SM more or less predicts the  $K^0$ - $\bar{K}^0$  mixing to the observed level. However, the  $\Delta m_K$  prediction is not so precise. The QCD improved short-distance contributions arising from the familiar box diagrams for the transition  $s\bar{d} \longrightarrow \bar{s}d$  lead to the following effective Hamiltonian [7]

$$\begin{aligned} \mathcal{H}_W^{Box} = & \frac{G_F^2}{4\pi^2} [\xi_c^2 S_0(x_c)\eta_1 + \xi_t^2 S_0(x_t)\eta_2 \\ & + 2\xi_c\xi_t S_0(x_c, x_t)\eta_3] (\bar{d}_L\gamma^\alpha s_L) (\bar{d}_L\gamma_\alpha s_L) + h.c. \end{aligned} \quad (10)$$

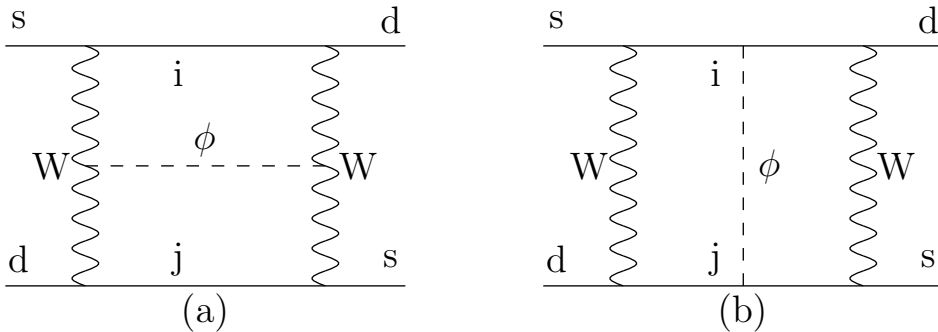
where  $\xi_i = V_{id}^* V_{is}$  is the CKM factor with the index  $i$  running over the quark flavours u, c and t and  $x_i = \frac{m_i^2}{M_W^2}$

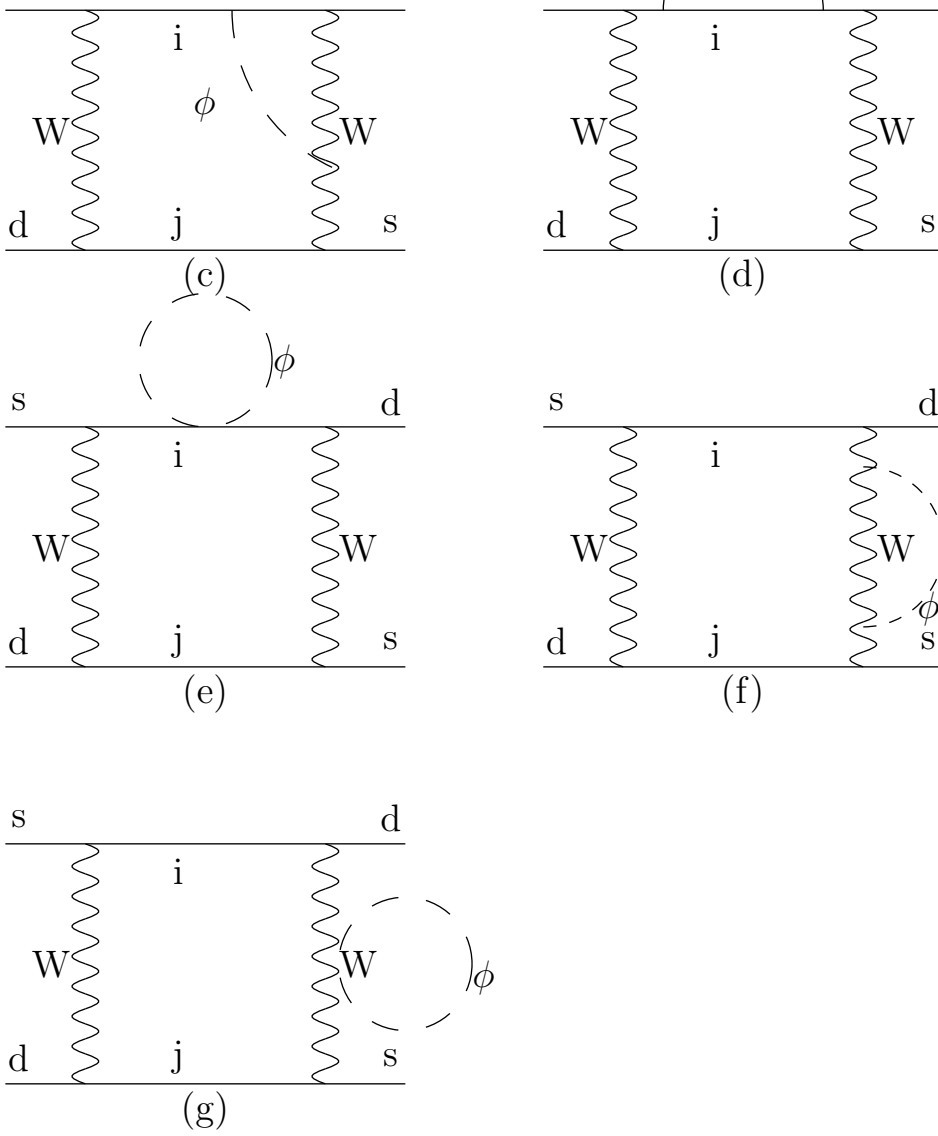
$$S_0(x) = \left[ \frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(1-x)^2} \right] - \frac{3x^2}{2(1-x)^3} \ln x \quad (11)$$

$$\begin{aligned} S_0(x, y) = & y \left[ -\frac{1}{y-x} \left( \frac{1}{4} + \frac{3}{2(1-x)} - \frac{3}{2(1-x)^2} \right) \ln x \right. \\ & \left. + (y \leftrightarrow x) - \frac{3}{4(1-x)(1-y)} \right] \end{aligned} \quad (12)$$

with  $\eta_i$  representing the QCD effects.

We now calculate the radion correction to  $\Delta m_K$ . The relevant box diagrams for the process  $s\bar{d} \longrightarrow \bar{s}d$  are





and their left-right or up-down reflections.

The calculations have been performed in the unitary gauge using dimensional regularization. The GIM mechanism ensures that the integrals are logarithmically divergent, thus justifying the use of dimensional regularization. The choice of the Unitary gauge avoids the inclusion of ghosts and additional corrections due to the interaction of the radion with the gauge fixing term, a point missed in [6]. The two-loop integrals (at zero external momenta) have been evaluated in  $d = 4 - \epsilon$  dimensions using the techniques similar to those given in [8]. It is found that the dominant contribution (of the form  $\frac{1}{\epsilon^2}$ ) comes from the diagrams (b), (c) and (d). The total leading contribution to the amplitude is

$$\begin{aligned} \iota T_{radion}(s\bar{d} \rightarrow \bar{s}d) = & \iota \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi \sin^2 \theta_w} \left( \frac{1}{16\pi^2} \right) \frac{3M_W^2}{8\langle\phi\rangle^2} \frac{1}{\epsilon^2} \\ & ( \bar{d}_L \gamma^\alpha s_L ) ( \bar{d}_L \gamma_\alpha s_L ) \sum_{i,j} x_i x_j \xi_i \xi_j \end{aligned} \quad (13)$$

where  $G_F$  is the Fermi constant,  $\alpha$  the fine structure constant,  $\theta_w$  the Weinberg angle. In the above expression, it is understood that  $\frac{1}{\epsilon}$  is to be replaced by  $\text{Log}\left(\frac{\Lambda^2}{m_\phi^2}\right)$ , where  $\Lambda$  is the cut-off scale of the theory. This results in the correction to  $\Delta m_K$  being given by

$$\Delta m_K^{radion} = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{6\pi} \frac{f_K^2 m_K}{\sin^2 \theta_w} \left(\frac{1}{16\pi^2}\right) \frac{3M_W^2}{8\langle\phi\rangle^2} \left[\text{Log}\left(\frac{\Lambda^2}{m_\phi^2}\right)\right]^2 \text{Re}\left(\sum_{i,j} x_i x_j \xi_i \xi_j\right) \quad (14)$$

with  $f_K$  and  $m_K$  being the Kaon decay constant and mass respectively.

As mentioned before, the observed size of  $\Delta m_K$  ( $(3.489 \pm 0.009) \times 10^{-12}$  MeV) [9] is roughly compatible with that expected in SM. Therefore, it would be natural to constrain the parameters of the theory (radion mass and VEV) by imposing the condition

$$\Delta m_K^{radion} \leq 0.009 \times 10^{-12} \text{ MeV} \quad (15)$$

This equation imposes an inequality on the space spanned by  $m_\phi$  and  $\langle\phi\rangle$ . constraining the *allowed region* (the region in the  $m_\phi$ - $\langle\phi\rangle$  plane above the curve in Fig.1).

## Appendix

Here we outline the steps that lead to the expression for interaction Hamiltonian as in equation(7) starting from the Lagrangian given in equation(6). Denoting the momentum conjugate to the fermion field  $\psi$  by  $\Pi_\psi = \frac{\partial \mathcal{L}}{\partial(\partial_0 \psi)}$  we have from equation(6)

$$\Pi_\psi = e^{\frac{-3\phi}{2\langle\phi\rangle}} \psi^\dagger \quad (16)$$

which is different from the corresponding flat space-time expression by the multiplicative warp factor while the momentum conjugate to the gauge field,  $\Pi_A$ , is same as in the flat space-time case. The Hamiltonian is

$$\mathcal{H} = \Pi_\psi \partial_0 \psi + \Pi_A \partial_0 A - \mathcal{L} \quad (17)$$

with  $\mathcal{L}$  as given in equation(6) and the Lorentz indices for the gauge field and its momenta suppressed. The interaction Hamiltonian is just the terms left after subtracting the free Hamiltonian from the above Hamiltonian. Making a canonical transformation to the interaction representation, the fields and momenta are

changed into  $\psi_{in}$ ,  $A_{in}$ ,  $\Pi_{Ain}$  and  $\Pi_{\psi_{in}} = \nu\psi_{in}^\dagger$ . Therefore, in the interaction representation (dropping the subscript  $in$ ),

$$\mathcal{H}_{int} = e^{\frac{-\phi}{2\langle\phi\rangle}} (m\bar{\psi}\psi) - e^{\frac{-\phi}{2\langle\phi\rangle}} \left(\frac{1}{2} M^2 A^\mu A_\mu\right) - g \bar{\psi}\gamma^\mu A_\mu\psi \quad (18)$$

which is exactly the same as obtained by rescaling the fermion field.

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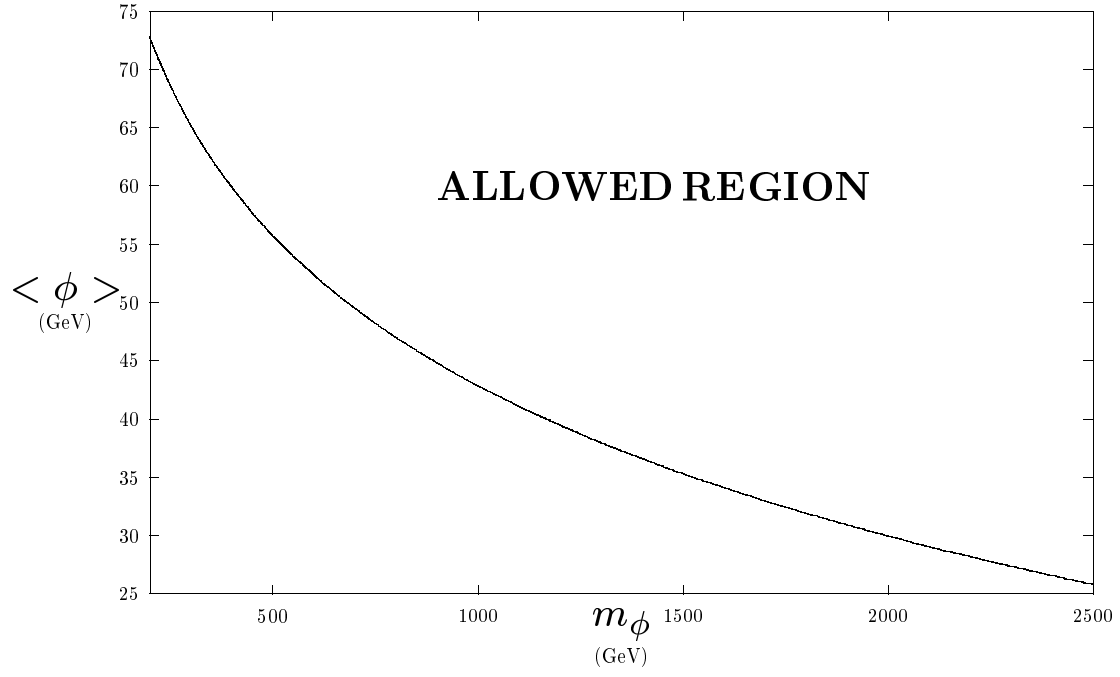


Fig.1: The allowed region for the vacuum expectation value and mass of the radion for cut-off  $\Lambda = 10$  TeV.